

AngLegs Unit - Family Of Triangles

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Topic / Subject: Planar Geometry - Family of Triangles

Objectives: Students will learn about the different types of triangles and explore properties of triangle angles.

Age: 5 years and up depending on the exercise

National Standards:

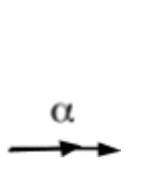
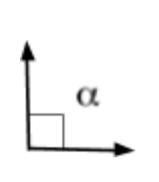
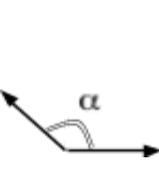
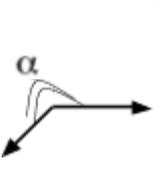
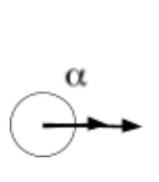
- CCSS. Math. Content. K. G. A. 1 & 2 - Identify and describe shapes.
- CCSS. Math. Content. K. G. B. 4 & 5 - Analyze, compare, create, and compose shapes.
- CCSS. Math. Content. 1. G. A. 1 & 2 - Reason with shapes and their attributes.
- CCSS. Math. Content. 2. G. A. 1 - Reason with shapes and their attributes.
- CCSS. Math. Content. 4. G. A. 2 - Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
- CCSS. Math. Content. 5. G. B. 3 & 4 - Classify two-dimensional figures into categories based on their properties.

Materials: AngLegs sticks and protractors, Grid paper, Pencil, Eraser, Ruler, Scissors, plastic straws, pipe cleaners.

AngLegs: AngLegs pack is composed of sets of 6 colors and clearview protractors. Each color of sticks represents a different length.

Background Information: Elementary Planar Geometry, Montessori Geometry Sticks

Prerequisites: Angles and types of angles (null or zero, acute, right, obtuse, straight, reflex, and complete or full angles), measuring of angles with a protractor

Angle type	Zero or Null Angle	Acute Angle	Right Angle	Obtuse Angle	Straight Angle	Reflex Angle	Full or Complete Angle
Angle value	$\alpha = 0^\circ$	$0^\circ < \alpha < 90^\circ$	$\alpha = 90^\circ$	$90^\circ < \alpha < 180^\circ$	$\alpha = 180^\circ$	$180^\circ < \alpha < 360^\circ$	$\alpha = 360^\circ$
Angle graph							

Definitions:

- * **Triangle** - Polygon of three sides, three vertices and three angles.
- * **Equilateral Triangle** - Triangle with three sides of equal length and three equal angles.
- * **Isosceles Triangle** - Triangle with only two sides of equal length and only two equal angles.
 - Acute Isosceles Triangle** - Isosceles triangle with three acute angles.
 - Right Isosceles Triangle** - Isosceles triangle with two acute angles and one right angle.
 - Obtuse Isosceles Triangle** - Isosceles triangle with two acute angles and one obtuse angle.
- * **Scalene Triangle** - Triangle with three sides of different lengths and three different angles.
 - Acute Scalene Triangle** - Scalene triangle with three acute angles.
 - Right Scalene Triangle** - Scalene triangle with two acute angles and one right angle.
 - Obtuse Scalene Triangle** - Scalene triangle with two acute angles and one obtuse angle.

Properties:

- * **Triangle Postulate** - The sum of the interior angles in every triangle is 180° (triangle postulate).
- * **Base Angle Theorem (Isosceles Triangle)** - If two sides of a triangle are congruent, the angles opposite these sides are congruent.
- * **Base Angle Converse (Isosceles Triangle)** - If two angles of a triangle are congruent, the sides opposite these angles are congruent.

Study of Types of Triangles:

What is a **triangle**? A triangle has three vertices and three sides.

1. Select 3 sticks of same color
Connect the ends into a 3-sided shape
Name this 3-sided polygon
Try with 3 sticks of a different color, how many different types of triangle can you make?
2. Select 2 sticks of the same color, and one of a different color
Connect the ends into a 3-sided shape
Name this 3-sided polygon
Changing the color of the pair of sticks and/or the third stick, how many different types of triangle can you make?
3. Select 3 sticks of the 3 different colors
Connect the ends into a 3-sided shape
Name this 3-sided polygon
Changing the colors of the sticks, how many different types of triangle can you make?
4. Defining attributes versus non-defining attributes:
Using plastic straws, cut them at different lengths and connect sets of three together using pipe cleaners. Make as many different triangles as possible.
Sort them by types of triangles. How many types did you get?
Does the color of the straws determine the types of triangle just made? Why?
What details define the types of triangle?

5. Place the straw triangles you made in the following table:

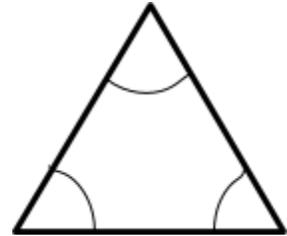
		S I D E S		
		Equilateral	Isosceles	Scalene
A N G L E S	Acute			
	Right			
	Obtuse			

- Find triangles in your classroom, identify the types of triangle they are and record them and their locations in your notebook.
- Find triangles in your school, identify the types of triangle they are and record them and their locations in your notebook.
- Using a ruler, and a protractor, draw the pictures of the seven types of triangles on construction paper. Cut them out and paste them in your notebook forming a large number 7.

Study of Types of Triangles: (Answer Key)

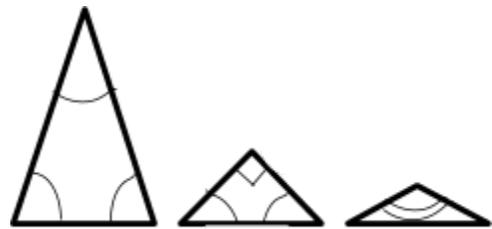
What is a **triangle**? A triangle has three vertices and three sides.

1. Select 3 sticks of same color
Connect the ends into a 3-sided shape
Name this 3-sided polygon
Try with 3 sticks of a different color, how many different types of triangle can you make?



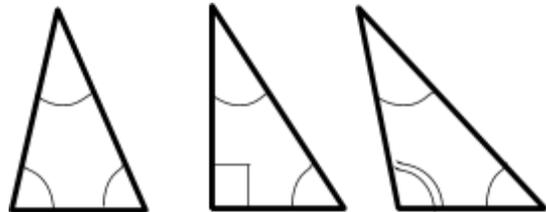
Only one type of triangle can be made with 3 sticks of the same color: an Equilateral Triangle.

2. Select 2 sticks of the same color, and one of a different color
Connect the ends into a 3-sided shape
Name this 3-sided polygon
Changing the color of the pair of sticks and/or the third stick, how many different types of triangle can you make?



Three types of triangle can be made with 2 sticks of the same color and 1 of a different color: Acute Isosceles Triangle, Right Isosceles Triangle and Obtuse Isosceles Triangle.

3. Select 3 sticks of the 3 different colors
Connect the ends into a 3-sided shape
Name this 3-sided polygon
Changing the colors of the sticks, how many different types of triangle can you make?

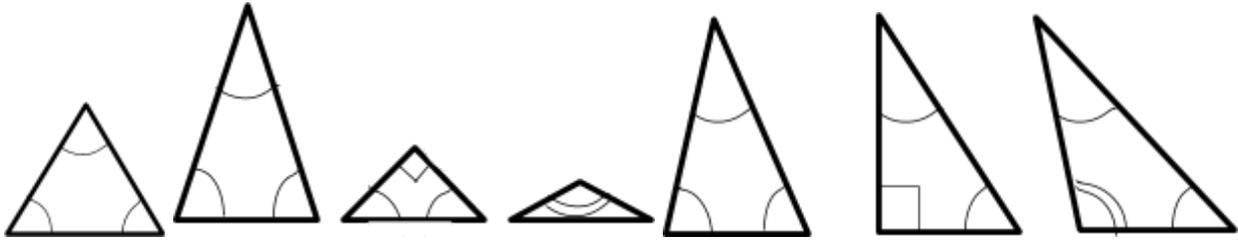


Three types of triangle can be made with 3 sticks of different colors: Acute Scalene Triangle, Right Scalene Triangle and Obtuse Scalene Triangle.

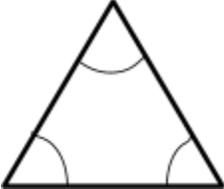
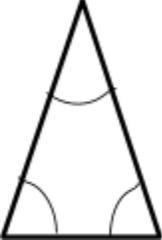
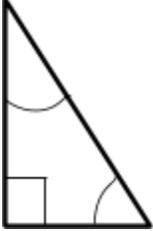
4. Defining attributes versus non-defining attributes:
Using plastic straws, cut them at different lengths and connect trios by pipe cleaners.
Make as many different triangles as possible.
Sort them by types of triangles. How many types did you get?
Does the color of the straws determine the types of triangle just made? Why?
What details define the types of triangle?

A total of seven types of triangle can be made: Equilateral Triangle, Acute Isosceles Triangle, Right Isosceles Triangle, Obtuse Isosceles Triangle, Acute Scalene Triangle, Right Scalene Triangle and Obtuse Scalene Triangle.

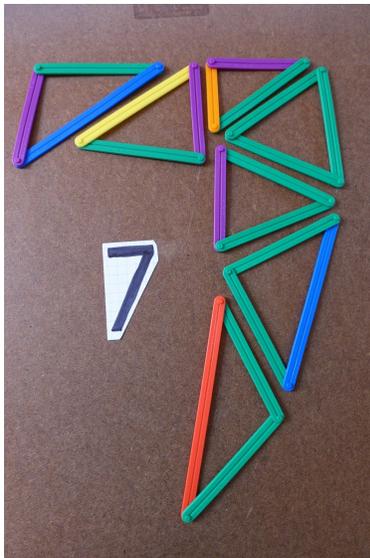
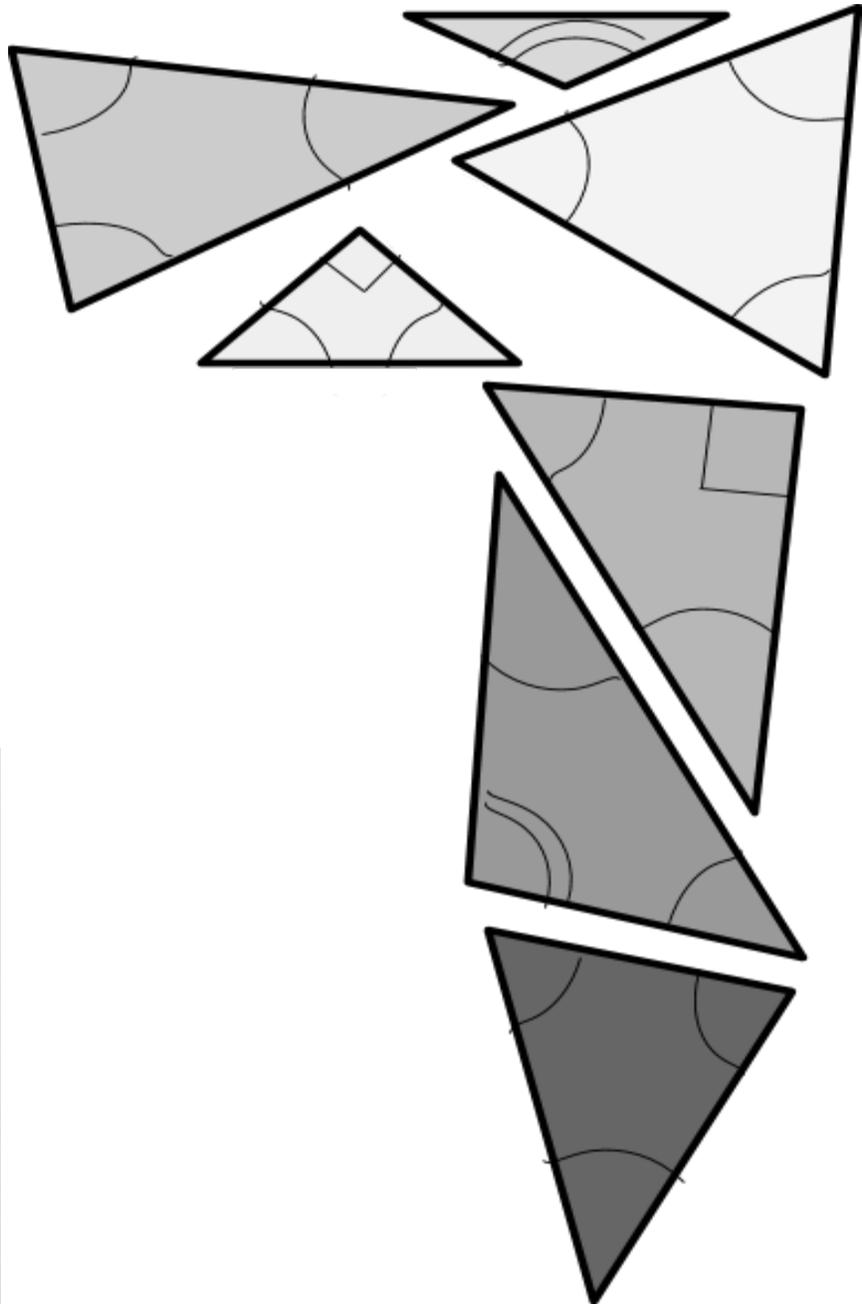
The color of the straws do not determine what type of triangle is made. The details of the triangle that identify its type are the lengths of its sides and its angles.



5. Place the straw triangles you made in the following table:

		S I D E S		
		Equilateral	Isosceles	Scalene
A N G L E S	Acute			
	Right			
	Obtuse			

6. Find triangles in your classroom, identify the types of triangle they are and record them and their locations in your notebook.
7. Find triangles in your school, identify the types of triangle they are and record them and their locations in your notebook.
8. Using a ruler, and a protractor, draw and label the pictures of the seven types of triangles in your notebook.



Study of Equilateral Triangles:

Study of equilateral Triangles:

- Select 3 sticks of same color
- Connect the ends into a 3-sided shape
- Name this 3-sided polygon
- Take a grid paper and trace the 3 inner angles
- Cut out the 3 inner angles
- Overlap the 3 angles on top of each other
- Compare the angles (similar or different)
- Using the protractor, measure and record the angle of each corner
- Arrange the 3 corners beside each other
- Observe, measure and record the sum of the 3 angles
- Reproduce activity with the other colors and compare.

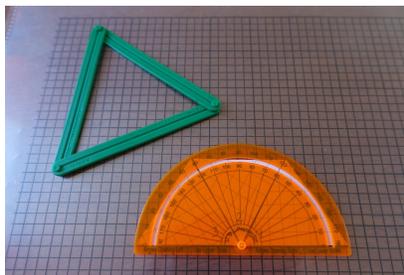
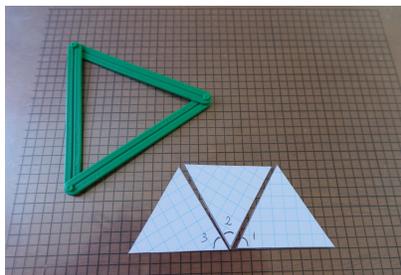
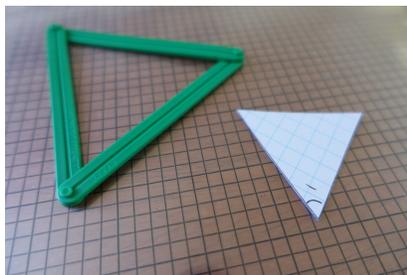
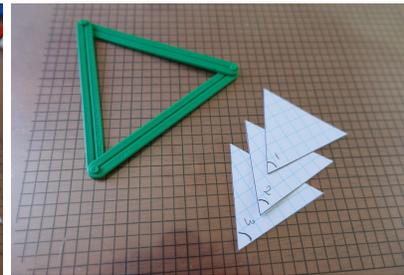
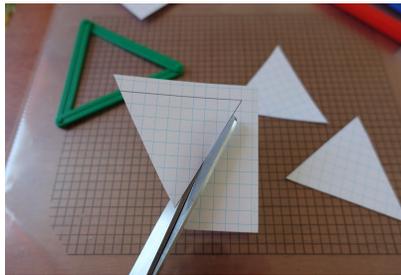
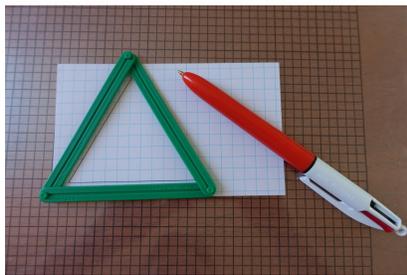
Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
3 Orange					
3 Purple					
3 Green					
3 Yellow					
3 Blue					
3 Red					

Study of Equilateral Triangles (Answer Key)

Study of Equilateral Triangles:

- Select 3 sticks of same color
- Connect the ends into a 3-sided shape
- Name this 3 sided polygon = **All 3 sides are of the same length, Equilateral Triangle**
- Take a grid paper and trace the 3 inner angles
- Cut out the 3 inner angles
- Overlap the 3 angles on top of each other
- Compare the angles = **All 3 angles are the same, Isogonal Triangle**
- Using the protractor, measure and record the angle of each corner = **60°**
- Arrange the 3 corners beside each other = **3 angles add to a straight angle**
- Observe, measure and record the sum of the 3 angles = **180°, straight angle**
- Reproduce activity with the other colors and compare.

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
3 Orange	60° acute	60° acute	60° acute	180° straight	Equilateral Triangle
3 Purple	60° acute	60° acute	60° acute	180° straight	Equilateral Triangle
3 Green	60° acute	60° acute	60° acute	180° straight	Equilateral Triangle
3 Yellow	60° acute	60° acute	60° acute	180° straight	Equilateral Triangle
3 Blue	60° acute	60° acute	60° acute	180° straight	Equilateral Triangle
3 Red	60° acute	60° acute	60° acute	180° straight	Equilateral Triangle



Study of Isosceles Triangles:

Study of Isosceles Triangles:

- Select 2 sticks of the same color, and one of a different color
- Connect the ends into a 3-sided shape
- Name this 3-sided polygon
- Take a grid paper and trace the 3 inner angles
- Cut out the 3 inner angles
- Overlap the 3 angles on top of each other
- Compare the angles (similar or different)
- Using the protractor, measure and record the angle of each corner, label the type of angle
- Arrange the 3 corners beside each other
- Observe, measure and record the sum of the 3 angles
- Reproduce activity with the other colors and compare.

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
2 Orange 1 Purple					
2 Orange 1 Green					
2 Orange 1 Yellow					
2 Orange 1 Blue					
2 Orange 1 Red					

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
2 Purple 1 Orange					
2 Purple 1 Green					
2 Purple 1 Yellow					
2 Purple 1 Blue					
2 Purple 1 Red					

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
2 Green 1 Orange					
2 Green 1 Purple					
2 Green 1 Yellow					
2 Green 1 Blue					
2 Green 1 Red					

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
2 Blue 1 Orange					
2 Blue 1 Purple					
2 Blue 1 Green					
2 Blue 1 Yellow					
2 Blue 1 Red					

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
2 Red 1 Orange					
2 Red 1 Purple					
2 Red 1 Green					
2 Red 1 Yellow					
2 Red 1 Blue					

Study of Isosceles Triangles: (Answer Key)

Study of Isosceles Triangles:

- Select 2 sticks of the same color, and one of a different color
- Connect the ends into a 3-sided shape
- Name this 3-sided polygon = **2 sides are of the same length, Isosceles Triangle**
- Take a grid paper and trace the 3 inner angles
- Cut out the 3 inner angles
- Overlap the 3 angles on top of each other
- Compare the angles (similar or different) = **2 angles are equal while the third is different**
- Using the protractor, measure and record the angle of each corner
- Arrange the 3 corners beside each other = **3 angles add to a straight angle**
- Observe, measure and record the sum of the 3 angles = **180°, straight angle**
- Reproduce activity with the other colors and compare.

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
2 Orange 1 Purple	90° right	45° acute	45° acute	180° straight	Right Isosceles Triangle
2 Orange 1 Green	120° obtuse	30° acute	30° acute	180° straight	Obtuse Isosceles Triangle
2 Orange 1 Yellow	180° straight	0° null	0° null	180° straight	Straight Line
2 Orange 1 Blue	Not possible	Not possible	Not possible	Not possible	Not possible
2 Orange 1 Red	Not possible	Not possible	Not possible	Not possible	Not possible

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
2 Purple 1 Orange	42° acute	69° acute	69° right	180° straight	Acute Isosceles Triangle
2 Purple 1 Green	76° acute	52° acute	52° acute	180° straight	Acute Isosceles Triangle
2 Purple 1 Yellow	90° right	45° acute	45° acute	180° straight	Right Isosceles Triangle
2 Purple	120°	30°	30°	180°	Obtuse

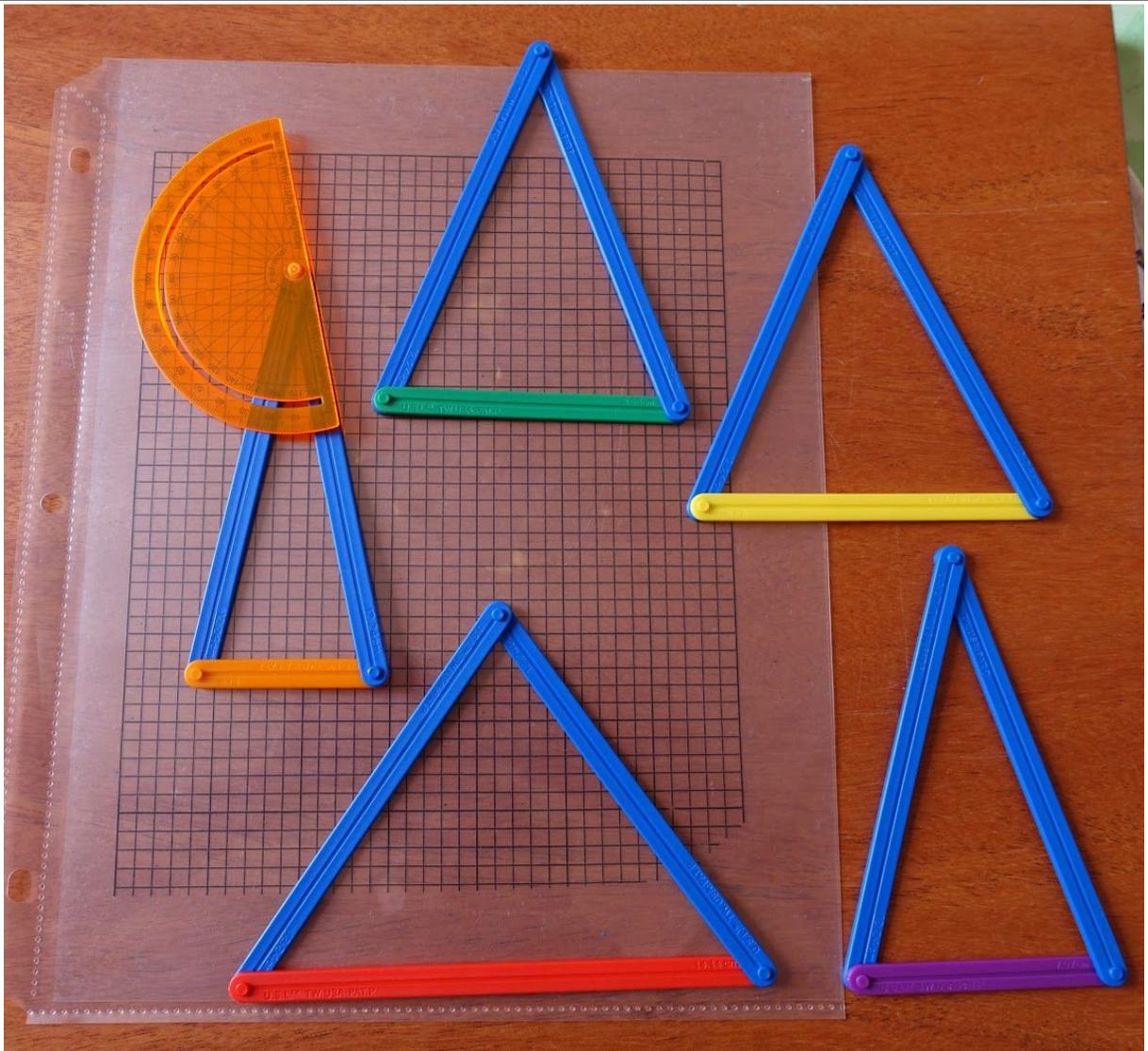
1 Blue	<i>obtuse</i>	<i>acute</i>	<i>acute</i>	<i>straight</i>	<i>Isosceles Triangle</i>
2 Purple 1 Red	180° <i>straight</i>	0° <i>null</i>	0° <i>null</i>	180° <i>straight</i>	<i>Straight Line</i>

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
2 Green 1 Orange	34° <i>acute</i>	73° <i>acute</i>	73° <i>acute</i>	180° <i>straight</i>	<i>Acute Isosceles Triangle</i>
2 Green 1 Purple	48° <i>acute</i>	66° <i>acute</i>	66° <i>acute</i>	180° <i>straight</i>	<i>Acute Isosceles Triangle</i>
2 Green 1 Yellow	71° <i>acute</i>	54.5° <i>acute</i>	54.5° <i>acute</i>	180° <i>straight</i>	<i>Acute Isosceles Triangle</i>
2 Green 1 Blue	90° <i>right</i>	45° <i>acute</i>	45° <i>acute</i>	180° <i>straight</i>	<i>Right Isosceles Triangle</i>
2 Green 1 Red	110° <i>obtuse</i>	35° <i>acute</i>	35° <i>acute</i>	180° <i>straight</i>	<i>Obtuse Isosceles Triangle</i>

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
2 Blue 1 Orange	25° <i>acute</i>	77.5° <i>acute</i>	77.5° <i>acute</i>	180° <i>straight</i>	<i>Acute Isosceles Triangle</i>
2 Blue 1 Purple	35° <i>acute</i>	72.5° <i>acute</i>	72.5° <i>acute</i>	180° <i>straight</i>	<i>Acute Isosceles Triangle</i>
2 Blue 1 Green	42° <i>acute</i>	69° <i>acute</i>	69° <i>acute</i>	180° <i>straight</i>	<i>Acute Isosceles Triangle</i>
2 Blue 1 Yellow	50° <i>acute</i>	65° <i>acute</i>	65° <i>acute</i>	180° <i>straight</i>	<i>Acute Isosceles Triangle</i>
2 Blue 1 Red	70° <i>acute</i>	55° <i>acute</i>	55° <i>acute</i>	180° <i>straight</i>	<i>Acute Isosceles Triangle</i>

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
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2 Red 1 Orange	20° acute	80° acute	80° acute	180° straight	Acute Isosceles Triangle
2 Red 1 Purple	29° acute	75.5° acute	75.5° acute	180° straight	Acute Isosceles Triangle
2 Red 1 Green	36° acute	72° acute	72° acute	180° straight	Acute Isosceles Triangle
2 Red 1 Yellow	42° acute	69° acute	69° acute	180° straight	Acute Isosceles Triangle
2 Red 1 Blue	51° acute	64.5° acute	64.5° acute	180° straight	Acute Isosceles Triangle



Study of Scalene Triangles:

Study of Scalene Triangles:

- Select 3 sticks of the 3 different colors
- Connect the ends into a 3-sided shape
- Name this 3-sided polygon
- Take a grid paper and trace the 3 inner angles
- Cut out the 3 inner angles
- Overlap the 3 angles on top of each other
- Compare the angles (similar or different)
- Using the protractor, measure and record the angle of each corner
- Arrange the 3 corners beside each other
- Observe, measure and record the sum of the 3 angles
- Reproduce activity with the other colors and compare.

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
1 orange 1 purple 1 green					
1 orange 1 purple 1 yellow					
1 orange 1 purple 1 blue					
1 orange 1 purple 1 red					
1 orange 1 green 1 yellow					
1 orange 1 green 1 blue					
1 orange 1 green 1 red					
1 orange 1 yellow 1 blue					
1 orange 1 yellow 1 red					

1 orange 1 blue 1 red					
1 purple 1 green 1 yellow					
1 purple 1 green 1 blue					
1 purple 1 green 1 red					
1 purple 1 yellow 1 blue					
1 purple 1 yellow 1 red					
1 purple 1 blue 1 red					
1 green 1 yellow 1 blue					
1 green 1 yellow 1 red					
1 green 1 blue 1 red					
1 yellow 1 blue 1 red					

Study of Scalene Triangles: (Answer Key)

Study of Scalene Triangles:

- Select 3 sticks of the 3 different colors
- Connect the ends into a 3-sided shape
- Name this 3-sided polygon = ***no sides are of the same length, Scalene Triangle***
- Take a grid paper and trace the 3 inner angles
- Cut out the 3 inner angles
- Overlap the 3 angles on top of each other
- Compare the angles (similar or different) = ***all 3 angles are different***
- Using the protractor, measure and record the angle of each corner
- Arrange the 3 corners beside each other = ***3 angles add to a straight angle***
- Observe, measure and record the sum of the 3 angles = ***180°, straight angle***
- Reproduce activity with the other colors and compare.

Stick colors	Angle #1 Type	Angle #2 Type	Angle #3 Type	Sum of Angles	Type of Triangle
1 orange 1 purple 1 green	35° acute	55° acute	90° right	180° straight	Right Scalene Triangle
1 orange 1 purple 1 yellow	28° acute	40° acute	112° obtuse	180° straight	Obtuse Scalene Triangle
1 orange 1 purple 1 blue	0° null	0° null	180° straight	180° straight	Straight Line
1 orange 1 purple 1 red	Not possible	Not possible	Not possible	Not possible	Not possible
1 orange 1 green 1 yellow	30° acute	60° acute	90° right	180° straight	Right Scalene Triangle
1 orange 1 green 1 blue	20° acute	35° acute	125° obtuse	180° straight	Obtuse Scalene Triangle
1 orange 1 green 1 red	Not possible	Not possible	Not possible	Not possible	Not possible
1 orange 1 yellow 1 blue	25° acute	53° acute	102° obtuse	180° straight	Obtuse Scalene Triangle
1 orange 1 yellow 1 red	16° acute	30° acute	134° obtuse	180° straight	Obtuse Scalene Triangle

1 orange 1 blue 1 red	21° acute	59° acute	100° obtuse	180° straight	Obtuse Scalene Triangle
1 purple 1 green 1 yellow	44° acute	58° acute	78° acute	180° straight	Acute Scalene Triangle
1 purple 1 green 1 blue	35° acute	44° acute	101° obtuse	180° straight	Obtuse Scalene Triangle
1 purple 1 green 1 red	24° acute	30° acute	126° obtuse	180° straight	Obtuse Scalene Triangle
1 purple 1 yellow 1 blue	35° acute	55° acute	90° right	180° straight	Right Scalene Triangle
1 purple 1 yellow 1 red	28° acute	42° acute	110° obtuse	180° straight	Obtuse Scalene Triangle
1 purple 1 blue 1 red	30° acute	60° acute	90° right	180° straight	Right Scalene Triangle
1 green 1 yellow 1 blue	45° acute	55° acute	80° acute	180° straight	Acute Scalene Triangle
1 green 1 yellow 1 red	38° acute	45° acute	97° obtuse	180° straight	Obtuse Scalene Triangle
1 green 1 blue 1 red	38° acute	60° acute	82° acute	180° straight	Acute Scalene Triangle
1 yellow 1 blue 1 red	44° acute	58° acute	78° acute	180° straight	Acute Scalene Triangle



Extension Activities:

1. Study other properties such as:
 - * **Sum of Exterior angles of polygons** - The sum of the exterior angles of a triangle is 360° .
 - * **Exterior Angle Theorem** - The exterior angle of a triangle is equal to the Sum of the 2 opposite interior angles.
2. Combining with the *Build & Learn Area & Perimeter kit* of the *Build And Learn Geometry STEM Kit* - **Geometry Proof of the Pythagorean Theorem $A^2 + B^2 = C^2$**
 - Pick a purple, a yellow, and a blue sticks from the *AngLegs kit* and connect them into a triangle. It forms a Right Scalene Triangle (as shown by placing a little tile in the right angle as shown in the first photo below left).
 - Place as many purple, blue and green tiles from the *Build & Learn Area & Perimeter kit* along the purple, blue and yellow sticks respectively. Count and record the amount of tiles.
 - Pythagorean Theorem: If A and B are the lengths of the legs of a right triangle and C is the length of the hypotenuse, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. $A^2 + B^2 = C^2$.
Make a purple, a green and a blue squares of tiles on the corresponding sides of the triangle. Count how many tiles for each color:
 - Overlap the green tiles over the blue ones and cover the rest by the purple tiles.
 - Observe how they overlap. What do you notice?
Calculate A^2 , B^2 , and compare to C^2 .

Standard CCSS.MATH.CONTENT.8.G.B.6 Understand and apply the Pythagorean Theorem as it is an explanation of a proof of the Pythagorean Theorem and its converse.

2. Combining with the *Build & Learn Area & Perimeter kit* of the *Build And Learn Geometry STEM Kit - Geometry Proof of the Pythagorean Theorem $A^2 + B^2 = C^2$*

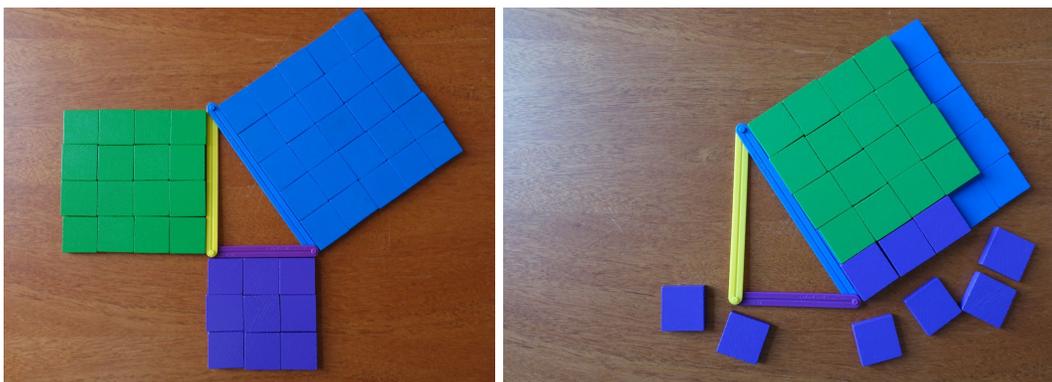
- Pick a purple, a yellow, and a blue sticks from the *AngLegs kit* and connect them into a triangle. It forms a Right Scalene Triangle (as shown by placing a little tile in the right angle as shown in the first photo below left).
- Place as many purple, blue and green tiles from the *Build & Learn Area & Perimeter kit* along the purple, blue and yellow sticks respectively like in the second photo below right. Count and record the amount of tiles.

Purple stick A = 3 purple tiles, yellow stick B = 4 green tiles, and blue stick C = 5 blue tiles for the hypotenuse.

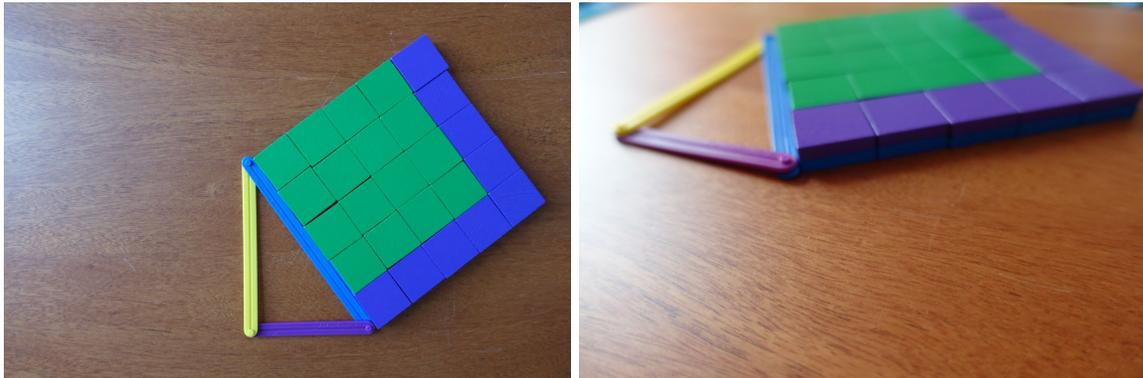


- Pythagorean Theorem: If A and B are the lengths of the legs of a right triangle and C is the length of the hypotenuse, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. $A^2 + B^2 = C^2$.
Make a purple, a green and a blue squares on the sides of the triangle as illustrated in the third photo below left. Count how many tiles for each color:

Purple stick $A^2 = 9$ purple tiles, yellow stick $B^2 = 16$ green tiles, and blue stick $C^2 = 25$ blue tiles for the hypotenuse.



- Overlap the green tiles over the blue ones and cover the rest by the purple tiles as presented in the fourth then in the fifth photos above right and below left.



- Observe how they overlap. What do you notice?

The 9 purple tiles of A^2 and the 16 green tiles of B^2 cover all the 25 blue tiles of C^2 proving that $A^2 + B^2 = C^2$: $3^2 + 4^2 = 5^2$ which is $9 + 16 = 25$.

Standard CCSS.MATH.CONTENT.8.G.B.6 Understand and apply the Pythagorean Theorem as it is an explanation of a proof of the Pythagorean Theorem and its converse.